Parametric Representation of Ground Antennas for Communication Systems Studies*

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Mathematical models relating the gain, cost, diameter, frequency, and rms surface tolerance of ground antennas are developed for both exposed and radome-enclosed parabolic reflectors. Diameters considered range from 15 to 500 feet, while frequencies vary from 1 to 100 GHz. Data from existing installations are used to develop standard cost vs diameter and rms surface tolerance vs diameter relationships. The standard cost is associated not only with diameter, but also with standard surface tolerance. Quality factors are introduced to relate deviations from the standard rms surface tolerances to expected departures from the standard cost curve. The models are completed by the inclusion of an approximate relation for the gain of parabolic reflectors. Each model comprises two equations among five variables. Although they are relatively complex, these models should be valuable to the communication systems planner in considering the gross features of alternative concepts for ground antenna installations. They are intended as guidelines for the conceptual stages of communication systems development, and are especially useful in terms of the trade-off studies they encompass. Three examples dealing with typical questions of this type illustrate their use.

I. INTRODUCTION

Mathematical models relating five important variables encountered in the consideration of ground antennas for communication systems are developed for both exposed and radome-sheltered structures. The variables are: diameter, cost, gain, frequency, and rms surface tolerance. Often, in communication systems studies, the costs associated

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with a ground antenna are estimated by using a simple relationship based on antenna diameter alone. The results of our study have a similar utility, but are substantially broader in scope and have greater flexibility. They represent some of the major features of ground antennas for use in the preliminary phases of communication systems planning.

In spite of the refinements offered by the present models in comparison with previously available guidelines, parameters which are important in tbe design and operation of a specific ground antenna system are not included. For example, there is no provision for including antenna noise temperature. Hence, the comparison of concepts on the basis of signal-to-noise ratio is not possible. Similarly, the effect of aperture illumination and tbe side lobe levels to be expected are not considered. There are other factors which must be accounted for in the design process but do not appear in the present formuation. For any specific application, performance requirements are carefully defined and vary considerably depending on the intended use of the antenna. It remains a challenge to the antenna specialist to optimize his design in order to meet those performance requirements in the best possible way. The present models, which characterize antennas in terms of only a few of their gross features, cannot and should not be expected to apply at such levels of refinement.

In this study, diameters range from 15 to 250 feet for the exposed antennas and 30 to 500 feet for antennas enclosed by a radome. Frequencies vary from 1 to 100 GHz. Only conventional reflectors are included. No consideration is given to actively controlled surfaces, multiple antenna synthetic apertures, or other such concepts which may be important in the future.

The first relation introduced is Ruze's formula relating gain to diameter, frequency, and rms surface tolerance. Two additional equations which relate a standard rms surface tolerance to diameter and a standard cost to diameter are developed from information available on existing and proposed antenna installations. Both the standard rms and the standard cost relationsbips are based on the same specific data points within each class of antennas. The points used are believed to be consistent and span tbe diameter range of interest. The result of this interpretation is a standard cost relationship which depends not only on the diameter of the antenna, but also is related to its surface tolerance.

The specific correlation of cost, diameter and rms surface toler-

ance is a novel feature of the present approach. Finally, quality factors are introduced to associate departures from the standard rms surface tolerance with departures from the standard cost curve. The functional relations chosen to represent these factors are justified with largely heuristic arguments because the existing information does not permit a more precise determination. However, when the data that is available on cost and rms surface tolerance is adjusted to a common standard using the functions chosen, the resulting agreement is encouraging.

Each model consists of two equations among the five variables of interest. Although much more complex than the simple power law relationship often used to represent the cost vs diameter of ground antennas, these sets of equations contain considerably more information. Not only do they readily yield information about any specific case, but also they provide a starting point for various optimization studies. Three examples are given, dealing with maximum cost-effective antennas, minimum cost antennas given the gain and the frequency, and the variation of gain for a specified cost and frequency. The first two examples are examined for both exposed and enclosed antennas. These examples suggest others that also could be done.

II. THE PROBLEM

The most elusive part of this study was determining satisfactory rms surface tolerance vs diameter and cost vs diameter relationships for ground antennas. Many reports were studied and several personal contacts made in the attempt to assemble sufficient information to draw the necessary conclusions. In spite of this effort, the functional relations which are ultimately suggested to represent standard cost and standard rms surface tolerance variations with diameter remain substantially empirical. It is worthwhile to discuss some of the reasons.

The original hope was to process the available data on existing and proposed antenna structures by some statistically satisfactory technique in order to determine the most likely functional forms for the relations of interest. This approach was abandoned for two reasons. In the first place, the number of antennas for which there is reliable information available on cost and rms surface tolerance is too small to admit a satisfactory statistical treatment. Secondly, cost and rms surface tolerance are vague and hard-to-define concepts which admit differing interpretations in each case. The lack of any common stand-

ards for reporting these quantities makes it unrealistic to determine functional relations involving such quantities by formal manipulations of the available data.

It is not hard to understand the reasons for the ambiguities in the reported data. The problems involved in measuring the surface tolerance of a large paraboloidal reflector are difficult, and to perform such a measurement is expensive and time-consuming. Often user demands on high performance antennas are sufficient to prohibit measuring the surface tolerance in any sort of a statistically satisfactory way. The measurement techniques themselves can place constraints on the structure, such as zero zenith angle and benign environmental conditions, which are unrealistic in terms of operational requirements. The data reduction process to determine the tolerance figure reported may introduce extraneous variables or unsuspected biases influencing the result. In a few cases, the surface accuracy has been calculated by measuring the gain over a range of frequencies and then using Ruze's gain equation, in which the rms surface tolerance is assumed to be the only unknown. This seems to be a powerful and effective technique, but it supposes a knowledge of the aperture efficiency at each frequency, a quantity that is extremely difficult to determine independently.[1]

Further ambiguity is introduced by neglecting to define the important concepts carefully. The rms surface tolerance can be measured with respect to the best-fit paraboloid or to the original design contour. It often contains a systematic as well as a random component, which may or may not have been eliminated in the published value. It can, of course, be a deviation normal to the reflector surface or normal to the aperture plane, although the distinction is not especially significant for shallow reflectors. In a few cases, it is the maximum peak-to-peak deviations that are reported and an equivalent rms surface tolerance must somehow be found.

The resolution of the cost associated with existing and proposed antennas, while not encumbered with the technical problems of surface tolerance determination, is beset with other difficulties. A high performance antenna is a custom-made item. The price reflects necessary research and development, engineering, tooling, and fabrication costs which are difficult to determine precisely and which cannot be spread over a large number of units. The requirements of each situation must be dealt with separately. There are relatively few companies building such structures, and the competition is fierce. Pricing

information and guidelines are proprietary and are simply not available to an interested outsider.

A different sort of problem arises in the attempt to establish the costs of existing structures. A specific cost figure can generally be found for most of the antennas in operation today, but it is difficult to determine exactly what the reported number of dollars bought. There are numerous ancillary items with a ground antenna that may or may not be included: the electronics, feed structure, servo systems, data readout and transmission, power plants, land acquisition, support buildings, heating, lighting, ventilation, and so on. Seldom is the reported cost broken down in sufficient detail. Meaningful cost comparisons cannot be made without knowing which subsystems are included in the reported cost and which are not.

In view of such uncertainties regarding the costs and surface tolerances of existing structures, and the relatively small numbers of such high performance antennas in operation, it is clear that a statistical approach to determining functional relations among the variables of interest would be illusory. Instead, the standard cost vs diameter and rms surface tolerance vs diameter relations are established by considering only a few data points (3 for exposed antennas and 4 for antennas with a radome) which span the diameter range of interest and seem to form a consistent subset of the data available.

The outcome of this line of reasoning is a pragmatic and qualitative model consisting of various relations among the variables of interest. It is not strictly defensible on grounds of statistical rigor, in spite of the rather satisfying way in which the available data are shown to fit within its structure. It is certainly neither unique nor absolute. Its usefulness lies in its reflection of acknowledged trends and its capacity as a basis for comparisons, trade-offs, and various sorts of optimization studies on ground antenna systems at a relatively coarse level.

III. THE GAIN FORMULA

In 1952, Ruze suggested a formula for the on-axis gain of a reflector antenna.[2] This formula has been generally acclaimed and enjoys wide popularity in spite of the restrictive assumptions it incorporates. These assumptions were clearly restated by Ruze in his 1966 article.[1]

Ruze's formula states:

$$G \cong \eta\left(\frac{\pi D}{\lambda}\right)^2 \exp - \left(\frac{4\pi\epsilon}{\lambda}\right)^2. \tag{1a}$$

Here D is the reflector diameter, λ is the wavelength at the frequency of interest, ϵ is the rms deviation of the reflector surface from the best-fit paraboloid, and η is the aperture efficiency, a measure of the overall electronic properties of the antenna. D, λ and ϵ must be in consistent units.

The leading factor in Ruze's formula expresses the gain for a perfect reflector. The effect of deviations from a perfect paraboloid are contained in the exponential factor. No distinction is made between manufacturing inaccuracies and deflections of the reflecting surface resulting from environment. The gain of a given antenna, with a specified diameter and surface tolerance, first increases as frequency is increased. However, a point is reached at which the exponential factor takes over, and a further increase in frequency results in a decrease of the gain. The point at which the gain is a maximum is called the gain-limit point.

The same phenomenon can be noticed if the frequency is held fixed and the diameter is varied. The cause for a gain-limit point in diameter is not immediately apparent from equation 1a, but it occurs because the rms surface tolerance is also a function of diameter. Stack has pointed this out in his work,[3] and he gives curves of gain vs diameter for a number of frequencies.[4]

The aperture efficiency, η, includes the effect of nonuniform illumination, spill-over, aperture blockage, front-end losses in antenna electronics and other factors which contribute to degradation in performance. It specifically does not include the effects of an imperfect reflecting surface. For a properly engineered antenna, η should lie between 0.65 and 0.75 (see Ref. 5). The aperture efficiency also depends to a certain extent on antenna configuration. In addition, the aperture efficiency depends on operating frequency for a given reflector. As a result of the uncertainty associated with the aperture efficiency, when equation 1a is used in this report, the aperture efficiency is simply taken to be 70 per cent. Refinements of this assumption would require information that is not available.

Equation 1a requires modification in order to use it for antennas with a radome. Experience with operation of radome-enclosed antennas has been generally satisfactory at microwave frequencies. At such frequencies the radome is responsible for approximately 1 dB loss in gain mostly from aperture blockage.[6] It also contributes to system noise temperature. The total system degradation depends to a considerable extent on local weather conditions. Consideration of

these effects is beyond the scope of this discussion. Further details can be found in Ref. 6.

There is little experience with radomes at millimeter frequencies. In this regime, the radome thickness is no longer small with respect to wavelength and special design techniques will clearly be necessary to minimize losses. For the present, we assume that equation 1a can be modified appropriately by means of a multiplicative factor

$$G \cong R(\lambda)\left[\eta\left(\frac{\pi D}{\lambda}\right)^2 \exp -\left(\frac{4\pi\epsilon}{\lambda}\right)^2\right]. \tag{1b}$$

The factor $R(\lambda)$ is chosen to represent the loss in gain caused by the presence of a radome, both because of aperture blockage and path losses in the radome.

The gain calculated using Ruze's formula (1a or 1b) does not include atmospheric effects that can degrade signal strength, such as turbulence or rain. These effects may be extremely important, particularly at high frequencies, but are not explicitly part of the ground antenna considerations.

IV. COST AND DIAMETER

When the information available on the costs of ground antennas is plotted with the diameters, a substantial scatter of the data is evident. Even with logarithmic coordinates, the dispersion precludes a satisfactory straight-line fit to the data points over the entire diameter range. Such a straight-line fit in logarithmic coordinates would correspond to the familiar power law relation, cost = (constant) (diameter)^n. A piecewise-linear cost function, corresponding to an increase in the power law exponent with diameter would be much better, but would introduce troublesome analytic complications.

To establish the standard cost vs diameter relationship, we have selected three antenna structures which span the diameter range of interest, and have fit a three-parameter expression to these three data points. The antennas chosen are:*

(*i*) The 15-foot antenna operated by Aerospace Corporation, El Segundo, California.[7]

(*ii*) The 85-foot antenna operated by the Naval Research Laboratory at Maryland Point, Maryland.[8]

* A typical reference is given for each antenna. There are several other sources describing each of the antennas.

(*iii*) The 210-foot antenna operated by the Jet Propulsion Laboratory at Goldstone, California.[9]

All three antennas were manufactured by the same company, and all are exposed and fully steerable, although the first two have polar mounts and the third has an azimuth-elevation mount. Good rms surface tolerance information is available for all three. More importantly, the cost information obtained from user and manufacturer agrees reasonably well for all three structures. The costs cited include the structure, drives, and control, but do not include electronics, readout equipment, or other ancillary costs, insofar as could be determined. The standard cost-diameter relation obtained this way is

$$\$^* = 6.7(10)^5 D^{-\frac{1}{3}} \exp\,(D/45). \tag{2a}$$

This curve is shown in Fig. 1. In equation 2a the diameter D is in feet. Potter's power law curve[10] for the 85- to 250-foot range is also shown in Fig. 1.

Although the relation 2a fits the three selected points very nicely,

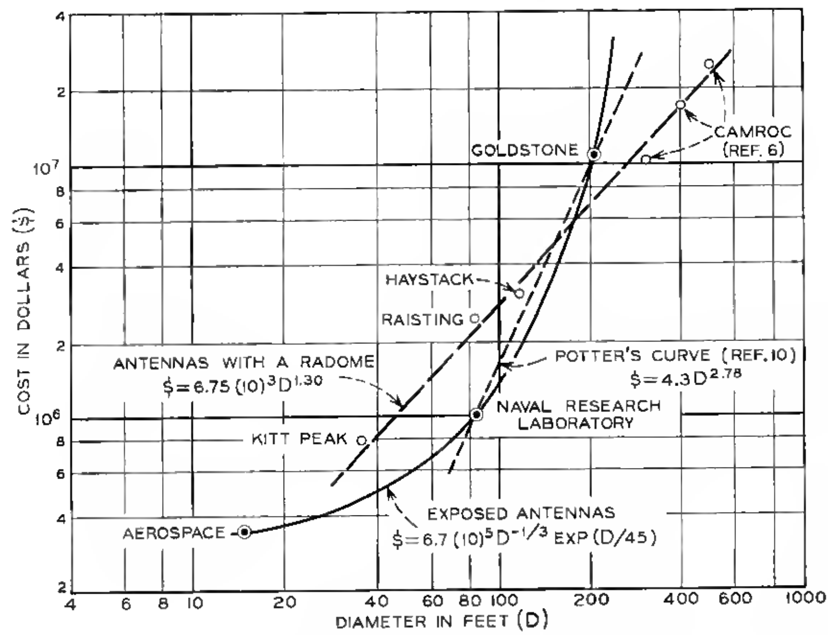

Fig. 1 — Antenna cost vs antenna diameter for both exposed and radome enclosed antennas (standard curves).

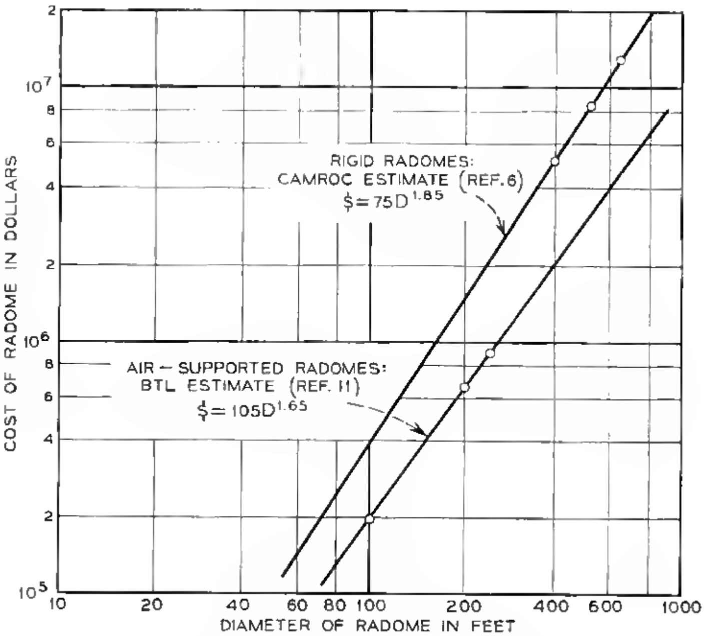

Fig. 2 — Radome cost vs radome diameter for rigid and air-supported radomes.

problems can occur if unconscious extrapolation is attempted. Beyond 210 feet, the costs increase very rapidly with diameter because of the exponential factor. There is a singularity at $D = 0$, and costs again increase as diameter decreases below 15 feet. The exponential increase in cost for very large antennas is probably not entirely unrealistic. However, relation 2a should be used only in a diameter range from 15 to 250 feet.

The cost-diameter relation for antennas with a radome is also shown in Fig. 1. In this case, the items included in, and excluded from, the reported cost are the same as for exposed antennas with one important exception. The cost of the radome is included. A simple power-law relation seems to be satisfactory for these antennas over a 30- to 500-foot diameter range. This relation is

$$\$^* = 6.75(10)^3 D^{1.30}. \tag{2b}$$

Again, the diameter is expressed in feet.

The estimated cost of the radome alone, including foundation and environmental control equipment, is shown in Fig. 2. Both air-sup-

ported and rigid space frame radomes are considered. This information is taken directly from Ref. 6 for the rigid radomes and from Ref. 11 for the air-supported radomes. Considerable extrapolation is required in both cases to cover the entire diameter range of interest. In addition, this data deals entirely with radomes designed to enclose antennas operating at relatively low frequencies. For the higher frequencies, special designs for the radome will have to be found in order to minimize losses and noise. Manufacturing and construction tolerances will be substantially more stringent than those reflected in the prices represented by Fig. 2. Radome cost will clearly depend on the operating frequency as well as on diameter. However, since there is virtually no experience with high frequency radomes, even the approximate nature of this dependence is unknown. In lieu of a more appropriate representation, the relations shown in Fig. 2 will be used in the present model.

The diameter of the radome required to enclose an antenna of diameter D is assumed to be $4/3\ D$. The cost of the antenna alone can now be determined using both Figs. 1 and 2.

V. SURFACE TOLERANCE AND DIAMETER

The data available on rms surface tolerance of existing antennas is plotted in Fig. 3. The ranges shown with many of the data points are attributable to a number of factors. In a few cases, they reflect honest uncertainty. In others, they represent a range of reported values resulting from the different tolerances at different elevation angles, or under different environmental conditions. In some cases the range shown encompasses values reported from different sources for the same antenna. For one or two antennas, the range shown represents the design goal.

The surface tolerance data shown represents the surface accuracy under operating conditions. Thus all factors that combine to produce mechanical deviations from a perfect paraboloid are included. These include manufacturing inaccuracies, as well as surface deflections caused by gravity, wind, and thermal effects. In general, the antennas represented are fully steerable, and operate satisfactorily in steady winds up to about 30 mph and other environmental conditions normally expected for such antennas.

The antennas used as a basis for the cost curves (Fig. 1) also determine the rms tolerance vs diameter curves. These curves are given by two straight lines; one for exposed antennas, and the other for

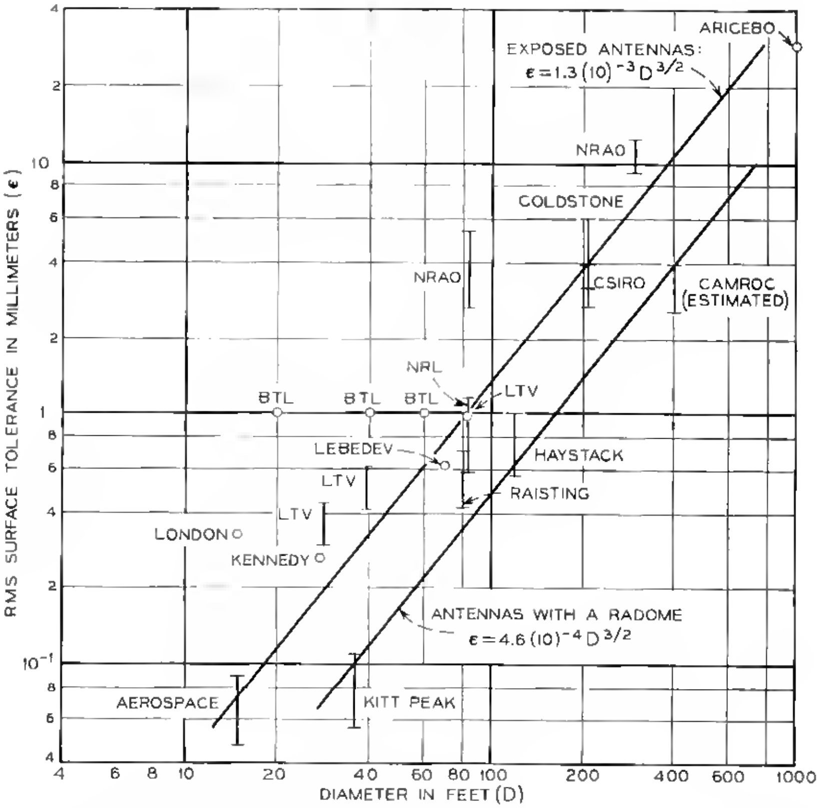

Fig. 3 — Antenna rms surface tolerance vs antenna diameter for both exposed and radome enclosed antennas (standard curves).

antennas operated inside a radome. The appropriate functional relationship is of the form:

$$\epsilon^* = \alpha D^{3/2} \tag{3}$$

where ϵ^* is the standard rms surface tolerance in millimeters, and D is the reflector diameter in feet. Caution: take special note of this rather unusual juxtaposition of units. The constant α is;

$$\alpha = 1.3(10)^{-3} \text{ for exposed antennas,}$$
$$\alpha = 4.6(10)^{-4} \text{ for antennas with a radome.}$$

The curve for reflectors protected by a radome has the same slope as the one for exposed structures but, at any given diameter, the surface

errors are considerably less for the enclosed structure because of the benign environment.

Since the same points have been used to develop the standard expressions, the cost-diameter relations (equations 2) give not just the cost of an antenna of diameter D, but specifically the cost of an antenna of diameter D with a surface tolerance given by equation 3. This correlation between the cost-diameter and rms-diameter curves is extremely important. The two relationships, taken together, express cost in terms of diameter and rms tolerance. The step from rms tolerance to frequency is simple, thus the cost is implicitly related to diameter and frequency. The costs and rms surface tolerances defined by these curves are called the standard values, and are indicated by the symbols ϵ^* and $\$^*$, respectively.

VI. QUALITY

We now turn to a determination of the effects of departures from the standard curves. For example, how much can be saved by relaxing the rms requirement at a given diameter? How much more will it cost to improve the surface tolerance at some given diameter? These questions are typical of many others, such as, is it better to increase diameter or surface tolerance to achieve desired performance?, and the possibilities of an interesting optimization study begin to emerge.

To deal with these questions, we introduce quality factors to relate departures from standard rms surface tolerance to departures from the standard cost. This ingenious approach to the problem was first introduced by Stack.[3, 4] The actual rms (ϵ) and actual cost ($\$$) can be expressed as

$$\epsilon = f_1 \epsilon^*,$$
$$\$ = f_2 \$^*$$

where ϵ^* and $\$^*$ represent the standard values as shown in Figs. 1 and 3. For antennas with radomes, the cost appearing in these relations is the cost of the antenna alone. The problem now reduces to an appropriate selection of the quality factors f_1 and f_2.

The functions chosen are

$$f_1 = 1/x \tag{4a}$$

$$f_2 = \exp(x - 1). \tag{4b}$$

The parameter x provides the connection between the quality factors. For $x > 1$, the actual rms surface error is less than the standard rms

error as given by equations 2. Conversely for $x < 1$, the surface is less precise than given by the standard relations.

The range of the parameter is $0 \leqq x \leqq \infty$. This range includes the possibility of achieving a nearly perfect reflecting surface by requiring x to be very large. Physically, of course, this is not possible. There exists a limiting tolerance, almost certainly a function of diameter, beyond which the surface accuracy can no longer be improved. Unfortunately, this limit is unknown. Equations 4 represent a compromise witb this situation. Althougb equation 4a admits the possibility of infinite improvement in rms surface tolerance, equation 4b associates an infinite cost with such an improvement. In fact, the cost factor f_2 expressed by equation 4b extracts a very heavy cost penalty for even modest rms surface tolerance improvement. In addition, expression 4b limits the possible reduction of cost to approximately $\frac{1}{3}$ of the standard cost, regardless of the reduction of quality of tbe reflecting surface.

The factor f_1 is the same as the one proposed by Stack.[4] However, the factor f_2 is significantly different. Tbese factors are based on the realization that the standard curves of Figs. 1 and 3 represent very good reflecting surfaces indeed. It is reasonable to expect it to be extremely expensive to improve the surface quality still further, while some saving should result if the standards of accuracy were relaxed. The particular factors chosen would probably not be applicable if we had selected the three basic antennas from which the standard cost curve is derived nearer the center of the spectrum of available products. However, the three points actually used represent a definite bias toward the excellent, and this bias justifies the present choice of the quality factors.

There is insufficient good raw data on the rms tolerance and cost of existing antennas to establish the quality factors directly. At least two reliable data points would be necessary for each of several different diameters in order to succeed. We would expect that the quality factors should also reflect the influence of diameter. However, the inclusion of this effect could not be justified on the basis of the available information.

It is possible to check approximately the quality factors chosen against the data that is available. Although the cost of each antenna considered is influenced by factors not included here (such as environmental considerations and tracking requirements), the comparison will be carried out on the basis of rms surface tolerance and diameter alone. A value of the parameter x can be determined by comparing

the actual and the standard rms values according to Fig. 3 and using the definition of f_1. The appropriate cost factor f_2 can then be found from equation 4b. If the inverse of the calculated cost factor is applied to the reported cost of the antenna, a revised standard cost is found. This figure represents the expected cost of the antenna, had it been built to the standard rms surface tolerance. The points obtained by performing this exercise for several different antennas are shown in Fig. 4. While agreement is not perfect, it is considerably better than any possible fit to the unmodified raw data.

Of course, the quality factors given in equation 4 are not unique. Other functions can be found which provide the same sort of qualitative trends. Functions can be suggested which permit finite limits to be placed on the possible improvement or degradation of the rms surface tolerance, and finite limits can be established for the associated cost as well. Such functions are more complicated than those actually chosen, and the implications of their use have not been investigated. Presently available information simply does not permit a definite choice to be made among all the possible functions that might be appropriate. The decision was to accept a set of functions that were both qualitatively reasonable and analytically convenient, as given by equation 4.

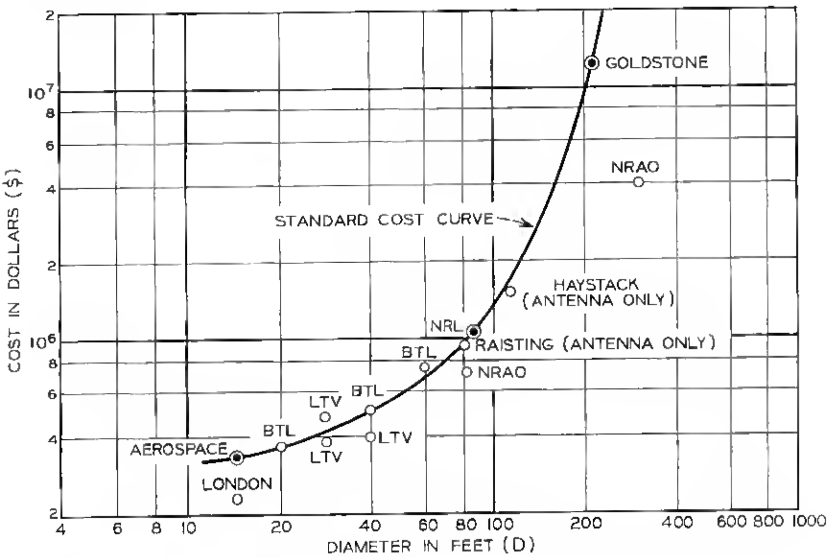

Fig. 4 — Revised standard cost vs diameter for existing antennas.

VII. MATHEMATICAL MODELS

Combining the equations developed above, we find the sets of equations which can be used to study the interrelation of gain, cost, diameter, surface tolerance and frequency, for both exposed and radome-enclosed antennas. For exposed antennas:

$$\epsilon = \frac{\alpha_1 D^{\frac{3}{2}}}{x}, \tag{5a}$$

$$\$ = \alpha_2 D^{-\frac{1}{2}} \exp(\alpha_3 D + x - 1), \tag{5b}$$

$$G = \eta(\alpha_4 D \Omega)^2 \exp -(\alpha_5 \epsilon \Omega)^2. \tag{5c}$$

Here, ϵ is the rms surface tolerance in millimeters, D the antenna diameter in feet, G the gain in absolute units, \$ the cost in dollars, η the aperture efficiency, and Ω the frequency in GHz, corresponding to a wavelength λ. Appropriate values for the constants are:

$\alpha_1 = 1.3 \times 10^{-3}$; $\alpha_2 = 6.7 \times 10^{5}$; $\alpha_3 = 2.22 \times 10^{-2}$;
$\alpha_4 = 3.20$; $\alpha_5 = 4.19 \times 10^{-2}$; $\eta = 0.70$.

Expressions 5 are appropriate for a diameter range of approximately 15 to 250 feet. For antennas with a radome:

$$\epsilon = \frac{\beta_1 D^{\frac{3}{2}}}{x}, \tag{6a}$$

$$\$ = \exp(x - 1)[\beta_2 D^{\beta_3} - \beta_4 D^{\beta_5}] + \beta_4 D^{\beta_5} \tag{6b}$$

$$G = R(\lambda)[\eta(\beta_6 D \Omega)^2 \exp -(\beta_7 \epsilon \Omega)^2] \tag{6c}$$

The terms have the same meaning as for exposed antennas. In the present model, the cost factor is applied only to the cost of the antenna. The total cost, however, includes the cost of the radome. The expression for gain has been modified by the factor $R(\lambda)$, which accounts for losses resulting from the radome. The system effect of the noise temperature contribution from the radome is not considered. Appropriate values for the constants in equation 6 are:

$\beta_1 = 4.6 \times 10^{-4}$; $\beta_2 = 6.75 \times 10^{3}$; $\beta_3 = 1.30$;
$\beta_6 = 3.20$; $\beta_7 = 4.19 \times 10^{-2}$; $\eta = 0.70$.
For rigid radomes: $\beta_4 = 1.28 \times 10^{2}$; $\beta_5 = 1.85$
For air-supported radomes: $\beta_4 = 1.69 \times 10^{2}$; $\beta_5 = 1.65$.

Expressions 6 are appropriate for a diameter range of approximately 30 to 500 feet.

VIII. EXAMPLES

Much information can be gleaned from the models expressed by equations 5 and 6. Each set is comprised of two expressions among five variables. (The parameter x can easily be eliminated between the first two equations of each set.) Thus if any three are specified the other two can be found directly. There are so many possible combinations that no general solution curves can be given. It is simpler to enter the appropriate equations for each specific case and work out the result.

Of more interest are the various optimization studies that can be carried out. We give the results of three specific studies here. These are obviously not the only such studies that could be done. The details of the necessary algebraic manipulations are omitted, since they are generally straightforward but often tedious. However, in each case the procedure is indicated. The discussion is phrased in terms of equation 5 for exposed antennas. However, the procedure described is also appropriate for equation 6, with obvious changes.

For all examples which include a radome, a rigid radome is assumed. In addition the attenuation factor, $R(\lambda)$, is set equal to 0.793. This corresponds to the assumption of a 1 dB loss caused by the radome, independent of frequency. While this is probably a reasonably good number to use for low frequencies, it is certainly an oversimplification for the higher frequencies in the range of interest. Results displayed as a function of frequency for antennas with radomes incorporate the implicit assumption that the radome is designed to match the operating frequency at each point. In other words, the results do not indicate the performance of a specific system as frequency varies, but imply that the radome design also varies to match the operating frequency.

8.1 *Example 1: Maximum Cost-Effective Antenna*

A maximum cost-effective antenna is defined as one which provides the most gain per dollar. By eliminating either the parameter x or the rms surface tolerance ϵ in the appropriate equations 5, both the cost and the gain can be expressed as functions of the diameter and the remaining variable, ϵ or x. The ratio $G/\$$ is formed, and maxima of this expression, considered as a function of two variables, are sought using standard techniques. $G/\$$ exhibits a single maximum in the frequency range of interest, and the location of the maximum depends on the frequency, as expected. The results of this example are

plotted in Figs. 5 and 6 for exposed and radome-enclosed antennas, respectively. The ordinates represent the diameter and the cost, plotted against a common abscissa, frequency. The gain at the point of maximum cost-effectiveness is indicated on the diameter curve.

There is a much greater variation of gain with frequency for exposed antennas than for antennas with radomes. This is directly attributable to the diameters involved. There is relatively little variation in cost for exposed antennas over the entire frequency range of interest. Such antennas cost about $500,000 regardless of the operating frequency. It is also notable that the maximum cost-effective antennas found in this example all operate well below their gain-limit point.

8.2 *Example 2: Minimum Cost Antenna for a Specified Gain and Frequency*

In this example, the operating frequency and the gain are specified, perhaps as a consequence of other system constraints. The first step

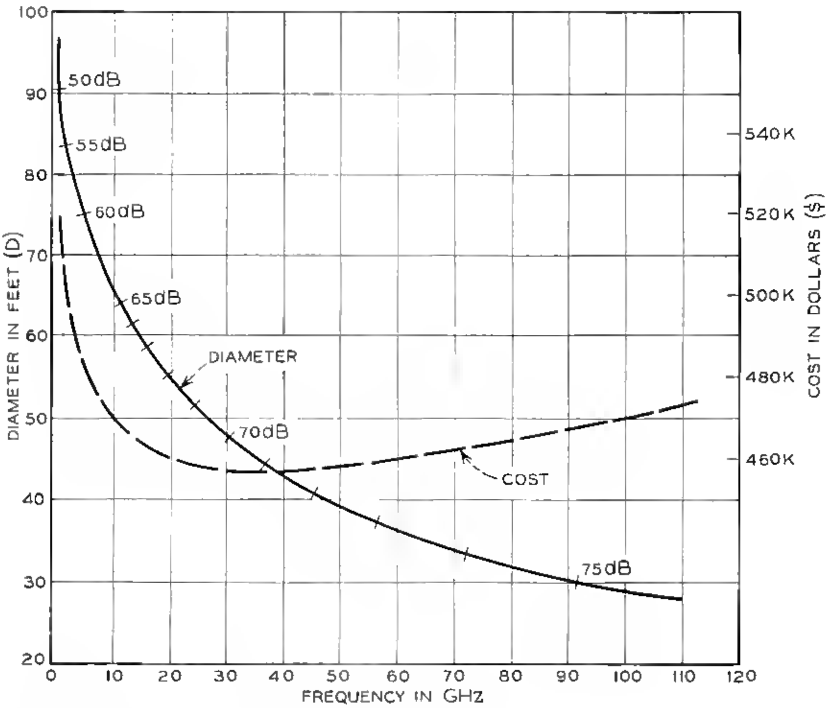

Fig. 5 — Cost and diameter of exposed maximum cost-effective antennas vs frequency (Example 1).

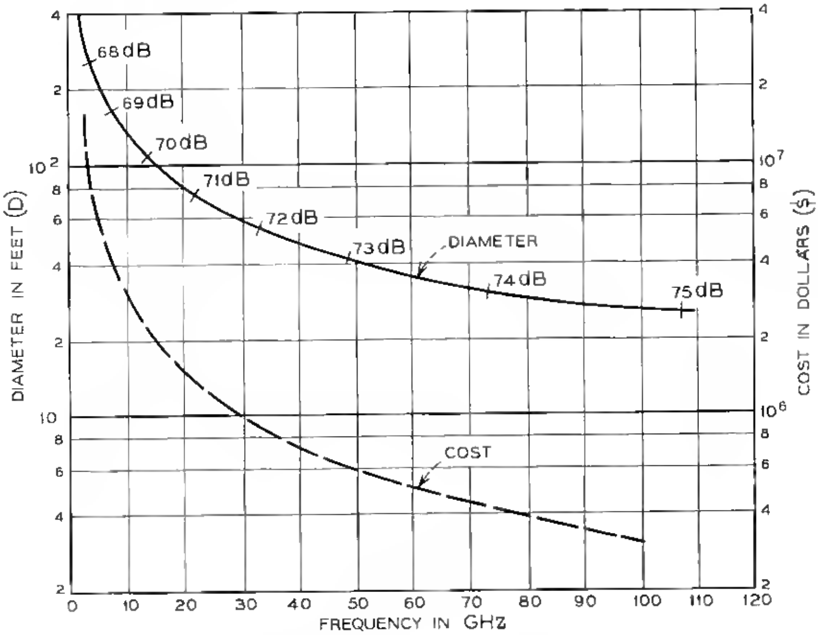

Fig. 6—Cost and diameter of radome enclosed maximum cost-effective antennas vs frequency (Example 1).

is to substitute equation 5a into equation 5c with G and Ω specified. The resulting expression can then be solved for the parameter $x(D)$. This expression for x is inserted in equation 5b, yielding cost as a function of diameter. The diameter which minimizes this cost is then found by differentiation. The algebra involved in this example is unpleasantly heavy and numerical search techniques were used to determine the minimum cost diameter for both the exposed and the radome-enclosed cases. These results appear in Figs. 7 and 8, respectively. Figure 7a shows diameter vs frequency, and 7b gives cost vs frequency, for several different values of gain. The same pattern is followed in Fig. 8.

For the radome-enclosed antennas, the results of this example are represented by straight lines in logarithmic coordinates. The diameter vs frequency relations for exposed antennas are also straight lines in logarithmic coordinates at the lower frequencies, but exhibit a definite curvature at higher frequencies, particularly for the lower gains. The cost vs. frequency curves for exposed antennas illustrate the excep-

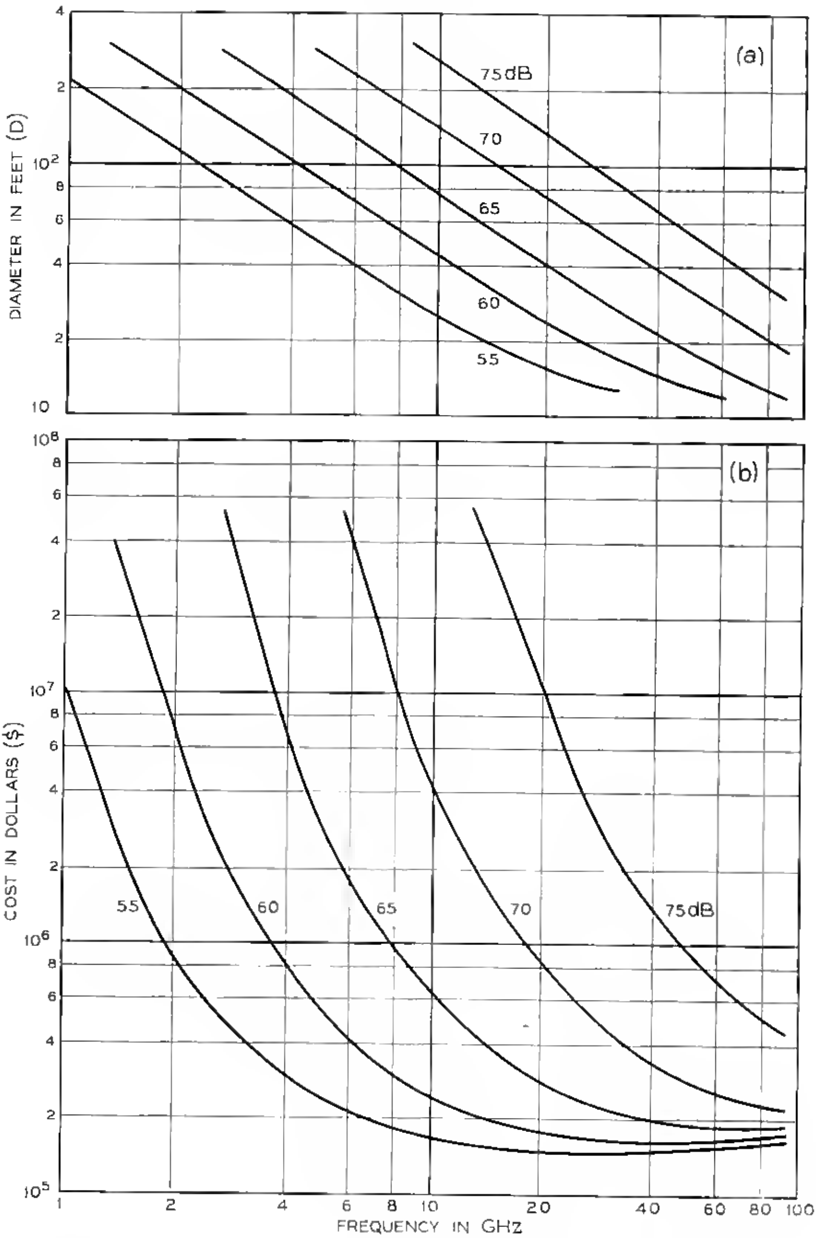

Fig. 7 — (a) Diameter and (b) cost of minimum cost exposed antennas for fixed gain and frequency vs frequency (Example 2).

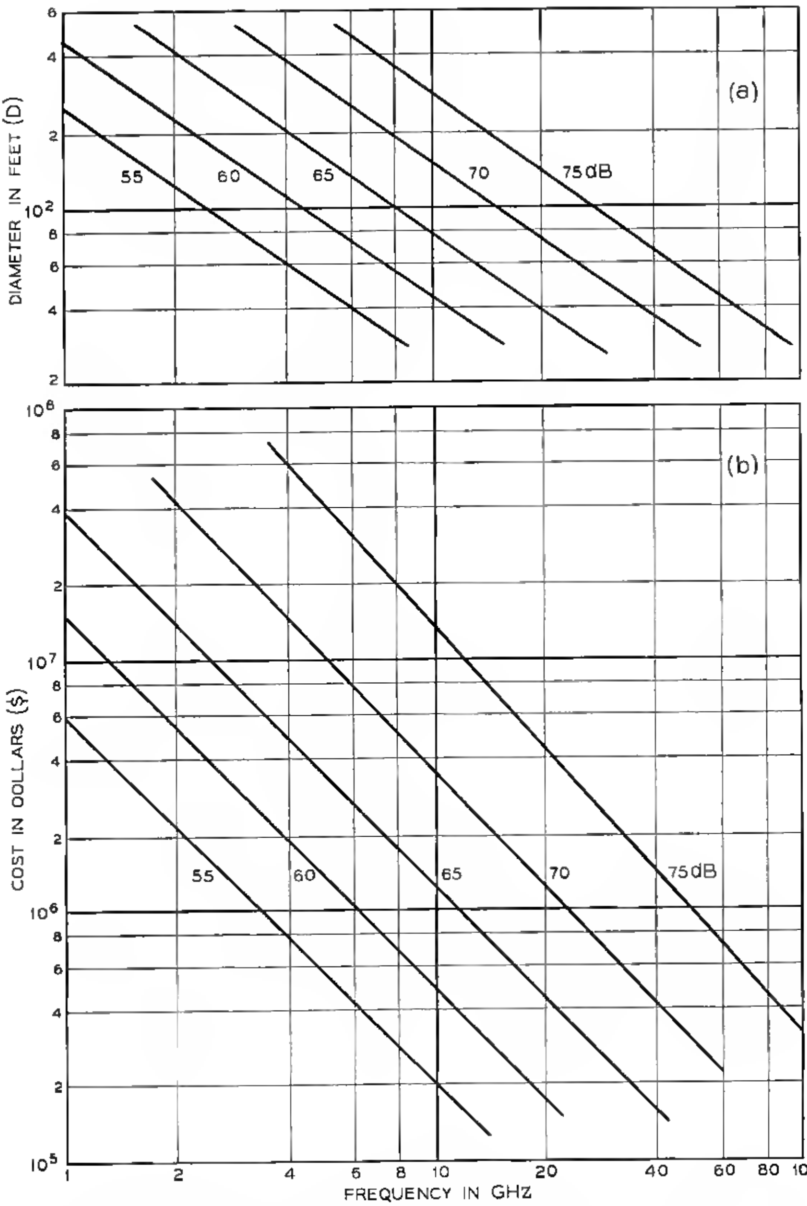

Fig. 8 — (a) Diameter and (b) cost of minimum cost radome enclosed antennas for fixed gain and frequency vs frequency (Example 2).

tionally high cost of gain at low frequencies because of the large diameter antennas required.

An interesting comparison can be made between the costs of equivalent exposed and enclosed systems by using Figs. 7b and 8b. For a specified gain, there is a range of diameters (or frequencies) within which an exposed antenna is less expensive than one enclosed in a radome. This range varies with gain. For other diameters (or frequencies) the radome enclosed system is a better buy for a specified performance.

8.3 *Example 3: Gain as a Function of Diameter for Fixed Cost and Frequency*

In this example, we do not consider the optimization possibilities directly, but we are interested in the gain as a function of diameter when the cost and the frequency are specified. For a given cost, x can be expressed in terms of the diameter according to equation 5b. In solving the model this way, we discovered that it was possible for x to be negative for certain combinations of cost and diameter. To avoid such a meaningless outcome, the restriction $x \geqq 0.1$ was imposed in this example. This is equivalent to restricting attention to antennas with rms surface tolerance no worse than 10 times the standard value. With $x(D)$ determined, equation 5a establishes the rms surface tolerance and the gain can be found from equation 5c with no difficulty.

Figure 9 shows the results of this exercise for two different fixed costs and several different frequencies. This figure shows the expected trends quite clearly. Notice that the optimum or gain-limit point for each set of conditions can be identified readily from the figure. This point could have been found directly, of course, by a procedure similar to that used in Example 2.

This example was not carried out for antennas with a radome.

IX. SUMMARY

Mathematical models relating cost, diameter, gain, rms surface tolerance and frequency have been developed for both exposed antennas and antennas with a radome. The form of the model in each case is a set of two equations among the five variables of interest. This model is much more complicated than others that have been suggested to relate the cost and diameter of ground antennas. However, it is also considerably more general and can be used to study a variety of possible trade-off situations. The major features of these models are:

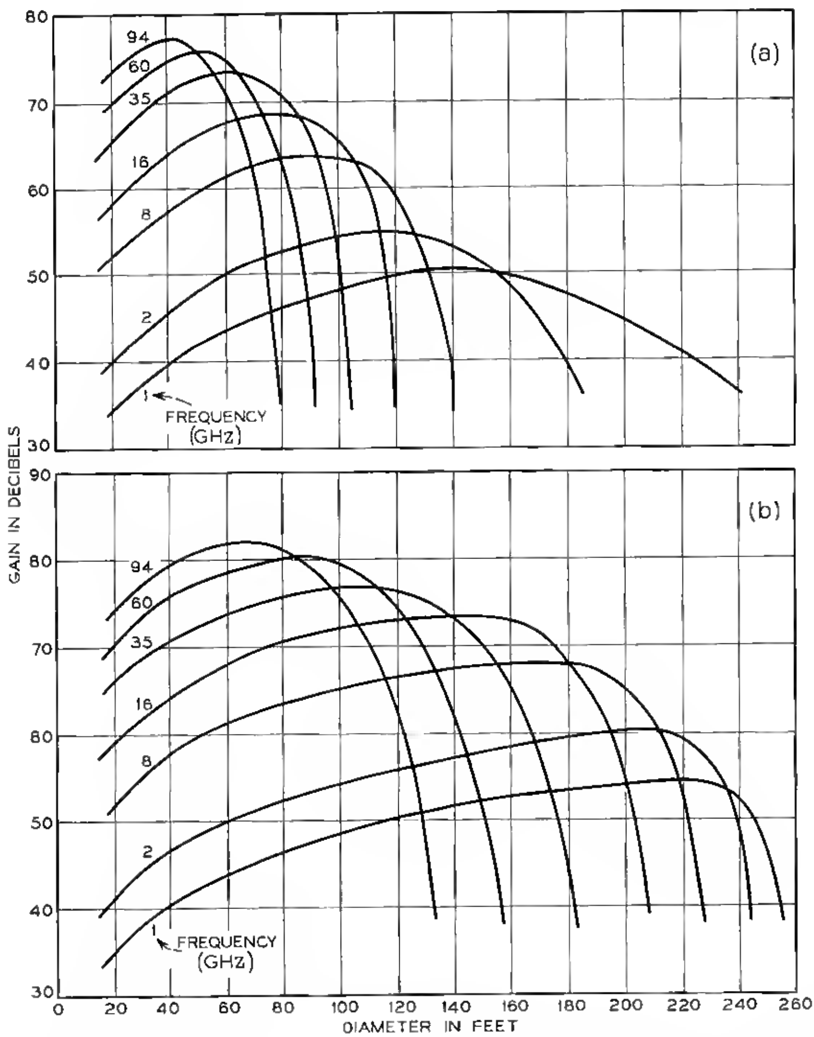

Fig. 9 — Gain vs diameter at several different frequencies with a fixed cost of (a) \$1,000,000, and (b) \$10,000,000 (Example 3).

(*i*) The inclusion of an exponential factor in the cost vs diameter relation for exposed antennas. This reflects, at least qualitatively, the exceptionally high cost associated with large, high performance exposed antennas.

(*ii*) The specific correlation of the cost vs diameter and rms sur-

face tolerance vs diameter relations. As a result, costs are associated not only with diameter, but also with rms surface tolerance.

(*iii*) The introduction of the quality factors. These factors relate deviations from the standard rms surface tolerance to expected departures from the standard cost curve. Although qualitative in nature, these factors reflect acknowledged trends. Skeptical readers who cannot accept the form of the quality factors used in the present models are encouraged to supply their own.

The two equations comprising each model can be supplemented by additional relations among the variables such as an rms surface tolerance-wavelength relation, for example. There is virtually no limit to the kinds of optimization studies and trade-off investigations that can be carried out within the framework of the suggested models. Examples of three such studies have been included. Through the credibility of the results, these examples further demonstrate the qualitative validity of the models.

These models have proved valuable in the preliminary phases of communications system planning, where competing concepts can be compared in terms of the relatively gross features of the system. They are not intended to usurp the responsibilities of the antenna designer in any specific application, and it would be erroneous to extrapolate their utility to such levels of refinement. Minor revisions in the constants of these models, as a result of new information or even different interpretations of present data, are to be expected and encouraged. However, such refinements should not invalidate the general applicability of the present models nor the qualitative conclusions drawn from their use.

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